Cooperative and Non-Cooperative Control in IEEE 802.11 WLANs

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Abstract: Numerous algorithms and techniques for optimal performance of an IEEE 802.11 WLAN have been investigated by researchers. These algorithms make use of either power control or PHY (physical layer) rate control (i.e., adaptive selection of PHY rates) or both to achieve maximum throughput levels for the network at minimum power consumption by the mobile devices. However most of these techniques are non-cooperative by definition (i.e., they attempt to maximize an individual node's performance and not the overall network performance).

In this report, we analyse cooperative and non-cooperative rate and power control based on an expression for the throughput of a node in an 802.11 WLAN that uses the Distributed Coordination Function (DCF) with an RTS/CTS frame exchange. We formulate a payoff function comprising of the throughput and costs related to power consumption of a mobile node. The payoff function is optimized and closed form expressions for the optimal PHY rate are obtained. In the cooperative approach we seek to obtain the optimal rates under two different scenarios – *max-min fair* rate and *global multirate* allocation. In the non-cooperative game approach we consider only *multirate* allocation. Our main contribution is that we obtain explicit expressions for the optimal PHY rate which in turn can be used to compute explicit expressions for the throughput. We consider optimization problems for both finite number of nodes n and for the limit $n \to \infty$. Single node throughputs corresponding to the optimal PHY rates are numerically studied and it is observed that network performance in the cooperative scenario is superior than in the non-cooperative scenario.

Key-words: 802.11, PHY rate, power, control, WLAN, modelling

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Contrôle Coopératif et Non-coopératif en WLAN IEEE 802.11

Résumé : De nombreux algorithmes et techniques pour la performance optimale d'un WLAN IEEE 802.11 ont été étudiés. Ces algorithmes se servent du contrôle de puissance ou du contrôle de débit PHY (couche physique) (c.-à-d. choix adaptatif des débits PHY), ou des deux pour maximiser le débit pour le réseau en minimisant la consommation d'énergie des dispositifs mobiles. Cependant, la plupart de ces techniques sont non-coopératives par définition (c.-à-d. elles essayent de maximiser la performance d'un noeud individuel et pas la performance globale du réseau).

Dans cet article, nous analysons le contrôle de débit et de puissance coopératif et non -coopératif en nous basant sur une expression pour le débit d'un noeud dans un WLAN 802.11 qui emploie la fonction 'Distributed Coordination Function' (DCF) avec un échange d'armature de RTS/CTS. Nous formulons une fonction de profit comportant le débit et les coûts liés à la consommation d'énergie d'un noeud mobile. La fonction de profit est optimisée et des expressions explicites pour le débit optimal de PHY sont obtenues. En approche coopérative, nous cherchons à obtenir les débits optimaux dans deux scénarios différents – débits *équitables max-min* et *multi-débit global*. Dans l'approche de jeu non-coopératif, nous considérons seulement le scénario *multi-débit*. Notre contribution principale est que nous obtenons des expressions explicites pour le débit optimal de PHY qui peuvent être également employées pour calculer des expressions explicites pour le débit. Nous considérons des problèmes d'optimisation pour un nombre fini de noeuds n ainsi que pour le cas $n \to \infty$. Les débits d'un noeud unique correspondant aux débits optimaux de PHY sont numériquement étudiées et nous observons que la performance du réseau dans le scénario coopératif est supérieure à la performance dans le scénario non-coopératif.

Mots-clés : 802.11, PHY rate, power, control, WLAN, modelling

1 Introduction

Mobile devices and Wireless local area networks (WLANs) based on the IEEE 802.11 technologies are becoming more and more popular. The technological challenge for these devices is to achieve maximum throughput levels constrained by the limited available power sources and operating physical data rates. While power is limited due to the limited amount of utilisable battery energy, operating physical data rates depend on the different modulation and channel coding schemes used. For example, the 802.11b PHY (physical layer) provides 4 PHY data rates ranging from 1 to 11 Mbps at the 2.4 GHz band and the high-speed 802.11a PHY, which has been developed to extend the 802.11 operation in the 5 GHz unlicensed band, provides 8 PHY data rates starting from 6 up to 54 Mbps.

Power control and PHY data rate control are two central mechanisms that are critical in achieving an efficient functioning of a WLAN. Power control and rate control are also very often used for providing quality of service (QoS) and both are useful for obtaining a radio channel with a low bit error rate (BER). In cellular networks, fixed rate services are often proposed, where power control is used to guarantee a required signal to interference ratio (SIR), see e.g. [11, 10, 3]. However, power control is not a *de facto* standard in IEEE 802.11a/b/g. Infact, the operating distance of 802.11a/b/g devices decreases with increase in the PHY rate being used. Thus additional power control is required. Only 802.11h which is being developed as an extension of 802.11a includes Transmission Power Control (TPC).

In this report, we analyse *cooperative* and *non-cooperative* power and rate control in an IEEE 802.11 WLAN environment. Our analysis is based on a throughput expression derived in [5]. The throughput expression has been validated in [22] using ns2 simulations ([22] is an extension of the work in [5] which in-turn is an extension of the work in [20]). We consider optimizing either the achieved aggregate network throughput (cooperative approach) or an individual node's achieved throughput (in a non-cooperative setup) by adaptively selecting one of the available PHY data rates. In the formulation of the optimization problems we further take into account a cost for power consumption. In our analysis we formulate a payoff function W_n for n users which is comprised of a utility part representing the throughput and a cost part related to power consumption. In the cooperative case the global payoff comprising of the total network throughput and total transmission power costs of all mobile nodes is maximized. In the non-cooperative game case, each player seeks to maximize its own payoff. The corresponding solution concept is then the Nash equilibrium.

By *non-cooperative* we mean here that each node attempts to use the transmission parameters (PHY rate and transmission power) by which it can maximize its own payoff (which can be the weighted sum of the throughput and power consumption) without any concern of the total aggregate network throughput and power consumption of the other mobile nodes. The notion of *cooperative* means that a common payoff (which can be taken as the sum of payoffs of all mobiles) is maximized; it represents the global throughput minus the global energy costs.

In the cooperative control analysis, we seek to maximize the payoff with two different approaches. In the first case, we seek to obtain an optimal fair assignment of PHY rates, with a *max-min* flavor, to all nodes irrespective of their channel conditions. (Of course, this means that a channel with bad conditions will have to use larger power). We then present a queueing model that allows us to compute the performance measures of interest associated with data transfers: expected transfer time and steady state probabilities. In the second approach, which we call the *global multirate* approach, we allow each node to use a different PHY rate and seek to obtain the optimal rate for each node. In this case, the optimal PHY rate used by each node will depend on its channel conditions. In addition to finite number of nodes, we also study the asymptotic regime in which there are a large number of nodes.

In the non-cooperative case, the goal is to maximize the individual payoff of each node. It has been shown by Tan et al. in [12] that in a non-cooperative setting under DCF (see next paragraph), individual nodes may intentionally transmit at lower data rates so as to achieve a higher throughput, but at the expense of a reduced overall throughput. Our analysis in this report confirms that in a non-cooperative setting optimizing an individual node's performance can cause overall network performance to suffer when compared to a cooperative setting. Indeed the DCF protocol as it is defined today does not give an optimal performance in a non-cooperative setting and can be improved to achieve higher overall network efficiency. Our main contribution in this report is that we obtain explicit expressions (or set of equations that can be solved numerically in the case of $n \to \infty$) for the optimal PHY rate. These explicit expressions are then used to calculate explicit throughput values to illustrate the inefficiency of DCF in a non-cooperative setting.

There are two different medium access control (MAC) protocols that have been specified for the IEEE 802.11 compliant devices. One is the contention-based Distributed Coordination Function (DCF) protocol, which is mandatory for all IEEE 802.11 devices and the other is the polling-based Point Coordination Function (PCF) protocol, which is optional. In DCF, which has been derived from the CSMA/CA protocol, contending nodes attempt to share channel resources by going into randomly chosen *back-off* durations (in units of *time-slots*) before carrying out a transmission. DCF does not require the presence of a central channel-resource allocating authority point and hence can be used in both *ad-hoc* and *infrastructure* networks. In PCF, access to the channel resources is centrally controlled by the Access Point (AP) and hence PCF can be used only in *infrastructure* networks. In this report, our analysis is based on the DCF protocol that uses the RTS/CTS frames before any data transmission attempts. Ergo, our discussion takes into account both *ad-hoc* and *infrastructure* networks.

1.1 Related Work

Application of power control in WLAN systems to minimize the required power in the *transmit mode* and adaptive selection of PHY rates has been studied by many researchers. In [13] the authors proposed a rate adaptation algorithm–Auto Rate Fallback (ARF) that was designed to optimize the application throughput. It is a non-cooperative algorithm in which each node in a WLAN attempts to selfishly use a higher transmission rate. Another well-known rate adaptation algorithm is Receiver based Auto Rate (RBAR) [14] whose goal is also to optimize the application throughput. It is again a non-cooperative rate control algorithm in which each node uses one of the available transmission rates depending on the Signal To Noise Ratio (SNR) calculated from the RTS/CTS frame exchange. In [15], the authors derive improved rate control based algorithms: Adaptive ARF and Adaptive Multi Rate Retry (AMRR), which are based on the ARF algorithm and a Multi Rate Retry mechanism. All these algorithms use only PHY rate control to achieve maximum throughput levels without considering any potential benefits that can be achieved by combining power control. Some other schemes have been proposed in [18] and [19] which incorporate only power control without considering the idea of an optimal PHY rate selection. The MiSer algorithm in [16] which is based on the 802.11a and 802.11h standards, is probably one of the few algorithms that combines the idea of PHY rate and power control. It provides an optimal combined rate-power tuple through a simple table-lookup. Each rate-power tuple is optimal in the sense of maximizing the energy efficiency (and not the throughput as in the algorithms mentioned previously). MiSer is also a non-cooperative attempt to obtain optimality by using combined rate and power control. In [17], the author has addressed the issue of joint link scheduling and power control for ad-hoc networks.

1.2 Motivation

Most of the algorithms or control schemes mentioned in Section 1.1 either consider only rate control or only power control to maximize the application throughput. Some other schemes like MiSer use both rate and power control to maximize the energy efficiency which is defined as the ratio of the expected delivered data payload to the expected total energy consumption. However, all these schemes are valid only for a non-cooperative environment. That is, they attempt to optimize an individual node's performance in terms of throughput or power consumption, as mentioned before. But optimizing an individual node's performance may cause the overall network performance to suffer.

Interestingly, Tan et al. in [12] have shown using both simulations and a game-theoretic approach that in a non-cooperative scenario under DCF, a "rational" node may achieve a higher throughput by using a lower transmission rate than by using a higher transmission rate, but at the expense of a reduced overall network throughput. This situation, as they explain, occurs due to the fact that in DCF each node has approximately equal probability of channel access. Thus by intentionally transmitting at a lower data rate, a "rational" node may achieve a higher channel occupation time than it would by transmitting at a higher data rate. Moreover, by transmitting at a lower data rate, the node reduces its BER. So the combined effect of higher channel occupation time and reduced BER may lead to a higher achieved throughput for that node, but at the expense of a poor overall network throughput. We will show later in Section 6 that a part of this result by Tan et al. can also be derived from our analysis.

Our analysis is motivated by the fact that a deeper insight into cooperative control is needed. The IEEE 802.11 specification for mobile devices allows customization of certain critical operation parameters like PHY data rate and MAC layer frame size. In a non-cooperative setting, these parameter settings can sometimes be misused by individual nodes to improve their performance at the expense of overall network performance (as it has been shown by Tan et al. in [12]). A detailed analysis of cooperative control can give insight into an alternate control mechanism and can answer some of the questions as to how misbehaving nodes in a non-cooperative setting can be prevented from adversely affecting aggregate network performance. Conversely, in an outdoor-picnic WLAN, when some friends sitting around in a park having identical channel conditions agree to use a common PHY rate for playing 'Quake-3' then we may wish to know what will be the optimal PHY rate that will maximize the overall network throughput at minimum power consumption as a function of the number of nodes/players or as a function of the MAC frame size. Note however that, if each player uses the maximum available PHY rate setting and not the optimal PHY rate then extra power may be consumed unnecessarily without giving any significant improvement in the overall payoff.

2 Model and background

Our analysis is based on the results obtained by Kumar et al. in [5]. Consequently, without going into much detail, we mention here a few assumptions (refer to [5] for details). Let there be n active nodes in a *single cell* IEEE 802.11 WLAN contending to transmit data. Each node uses the Distributed Coordination Function (DCF) protocol with an RTS/CTS frame exchange before any data-ack frame exchange and each node has an equal probability of the channel being allocated to it. It is assumed that every node has infinitely many packets backlogged in its transmission buffer. In other words, the transmission buffer of each node is *saturated* in the sense that there are always packets to transmit when a node gets a chance to do so. It is also assumed that all the nodes use the same *back-off* parameters. Let β denote the long run average attempt rate per node per *slot* ($0 < \beta < 1$) in *back-off time*[1] (conditions for the existence of a unique such β are given in [5]). We assume the *decoupling approximation* made by Bianchi in [20] which says that from the point of view of a given node, the number of attempts by the other nodes in successive slots are i.i.d. binomial random variables with parameters $(n-1)$ and β. Let the MAC frame size of node i be L_i bits and let the PHY rate used by this node be denoted by C_i bits per slot. Let T_o be defined as the transmission overhead in slots related to a frame transmission, which comprises of the SIFS/DIFS, etc and let T_c be defined as the fixed overhead for an RTS collision in slots. Then it follows from [5] that the throughput of node i is given by

$$\theta(i,n) = \frac{\beta(1-\beta)^{n-1}L_i}{1 + n\beta(1-\beta)^{n-1}\Big(T_o - T_c + \frac{1}{n}\sum_{i=1}^{n}\frac{L_i}{C_i}\Big) + \Big(1-(1-\beta)^n\Big)T_c} \tag{1}$$

where $\beta = \beta(n)$ (i.e., β is a function of n) is obtained as the solution of a fixed point equation that does not depend on L_i's or C_i's. The fixed point equation in [5] has been derived from the renewal reward theorem and uses the Binomial to Poisson convergence theorem. Note that by the throughput expression above, all nodes achieve the same single node throughput even if they use different rates. As is the case in IEEE 802.11, for all nodes that use an RTS/CTS frame exchange before the data-ack frame transmission, we assume throughout our discussion that

$$T_o \geq T_c \tag{2}$$

In our analysis in the following sections, we will consider optimization problems for both finite n and for the limit $n \to \infty$. For handling the latter case, we identify here the asymptotic aggregate throughput as $n \to \infty$ (this derivation can be found in [5] for the special symmetric case where all L_i's and C_i's are equal). An appealing feature of the asymptotic case is that we have an *explicit* expression for β.

Asymptotic throughput: Later in our discussion we will need the asymptotic throughput in the following two contexts:

[1] If we plot transmission attempts as a function of "real" time, and then *cut out* from the plot the channel activity periods (during which all mobiles freeze their back-off), then the new horizontal axis is called the "back-off time", see Section II.A of [5].

(i) In the max-min fair (MMF) case where we assign the same PHY rate to all mobile nodes, we consider all nodes to be symmetric, i.e., they all use the same PHY rate C and frame size L (they still may have different channel conditions). In this case, if first $K \to \infty$ [5] and then $n \to \infty$, the global throughput is given by Sec. VII.C in [5] as:

$$\tau(C) = \frac{L\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + \left(\frac{L}{C}\right)\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{3}$$

where p is the exponential back-off multiplier, i.e., if b_k is the mean back-off duration (in slots) at the kth attempt for a frame then $b_k = p^k b_0$. According to the IEEE 802.11 specifications $p = 2$.

(ii) In the case where we consider global multirate PHY rate assignment to all nodes, i.e., each node uses one of the c distinct available values of the parameters (C_i, L_i) with $(C_i, L_i) \in \{(C_1, L_1), ..., (C_c, L_c)\}$, we derive the corresponding asymptotic throughput below.

Assume that there are m_i nodes using parameters (C_i, L_i). Denote by $\alpha_i(n) = m_i/n$ the fraction of the nodes using (C_i, L_i) among all nodes in the cell. Then the throughput of all nodes using (C_i, L_i) is given by

$$\theta(\alpha_i(n)) = \frac{m_i \beta e^{-n\beta} L_i}{1 + n\beta e^{-n\beta}\left(T_o - T_c + \sum_{i=1}^{c} \frac{\alpha_i(n) L_i}{C_i}\right) + \left(1 - e^{-n\beta}\right) T_c} \tag{4}$$

where we use the Binomial to Poisson approximated version of the throughput expression for the asymptotic case mentioned in Section VII.C of [5]. It is assumed that $\alpha_i(n)$ converges to a limit α_i which is a probability measure. Note that the attempt rate $\beta = \beta(n)$ and the collision probability γ as defined in [5] are not functions of L_i nor of C_i. Thus, first taking $K \to \infty$ [5] and then taking the limit $n \to \infty$, it can be observed that

$$\lim_{n\to\infty} n\beta(n) \uparrow \ln\left(\frac{p}{p-1}\right) \tag{5}$$

where $\beta(n)$ is obtained as the solution of a fixed point equation corresponding to n nodes (see Theorem VII.2 in [5]). Combining (4) and (5) we get as $n \to \infty$ the following expression for the aggregate throughput of all nodes using (C_i, L_i):

$$\tau(\alpha_i) = \frac{\alpha_i L_i\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + \sum_{i=1}^{c}\left(\frac{\alpha_i L_i}{C_i}\right)\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{6}$$

Denote $E_\alpha[L/C] = \sum_{i=1}^{c} \frac{\alpha_i L_i}{C_i}$ and $E_\alpha[L] = \sum_{i=1}^{c} \alpha_i L_i$. Then it follows from Equation (6) that the asymptotic global throughput is given by

$$\tau(\alpha) = \frac{E_\alpha[L]\left(1-\frac{1}{p}\right)}{\frac{1}{\ln\left(\frac{p}{p-1}\right)} + \left(1-\frac{1}{p}\right)\left(T_o - T_c + E_\alpha[L/C]\right) + \frac{T_c}{p\ln\left(\frac{p}{p-1}\right)}} \tag{7}$$

3 Defining the payoff function

In an efficiently working WLAN, the goal of the mobile nodes is to achieve maximum throughput levels with minimized power consumption costs. In a cooperative scenario, the nodes should cooperate to achieve maximum overall network throughput at minimum combined power consumption. If each node uses the highest available PHY rate, which is say common for all nodes, it may not be the best strategy to achieve the most efficient overall network performance. The reason being that under the given channel conditions, a node may be unnecessarily consuming more power by transmitting at the highest available rate if transmitting at a lower PHY rate does not degrade the combined network throughput. Based on this thought and the fact that under DCF, each node has an equal probability of gaining access to the channel we define a long-term payoff function W_n for n active nodes in the WLAN as

$$W_n := \sum_{i=1}^{n} (\theta(i, n) - \zeta_i Q_i(C_i)) \tag{8}$$

where $\theta(i,n)$ is the throughput of node i as defined in Equation (1). $Q_i(C_i)$ is a cost related to the power consumption of node i and is a function of the PHY rate C_i and ζ_i is a weight that gives relative importance for node i to the cost versus the throughput. Note that, maximizing this payoff function leads to maximizing the throughput and minimizing the costs related to power consumption.

Experiments conducted by Gruteser et al. in [21] with IEEE 802.11 equipment reveal that under given channel conditions and a low transmission power range, the power consumed by a mobile node can be approximated as being linearly proportional to the PHY rate used. Consequently, $Q_i(C_i)$ can be considered as a linear cost function of the form:

$$Q_i^{lin}(C_i) = a_i C_i \tag{9}$$

where a_i is a random variable that may depend on the path attenuation under given channel conditions.

Next, motivated by the Shannon's theorem and assuming an AWGN channel that uses complex symbols , the transmission rate of a node is of the form

$$C(\pi) = W \log_2 \Big(1 + \frac{\pi}{z}\Big) \tag{10}$$

where W is the passband spectrum in *Hertz*. π is the transmission power of the node and $z = WN_o/h$, where N_o is the one-sided *power spectral density* of the channel noise and h is a random variable that characterizes the signal attenuation. z is therefore a random variable that may depend on the thermal noise in the channel. Equation (10) can be rewritten as:

$$\pi(C) = z\Big(e^{\psi C} - 1\Big). \tag{11}$$

where $\psi = \frac{ln2}{W}$. It has also been seen in the results of the experiments in [21] that the power consumed by mobile nodes is piecewise linearly proportional to the transmission power. Therefore, an exponential cost $Q_i^{exp}(C_i)$ can be assumed, which is of the form:

$$Q_i^{exp}(C_i) = z_i\Big(e^{\psi C_i} - 1\Big). \tag{12}$$

From the definitions of a_i and z_i in the foregoing discussion it is evident that their values may vary from one mobile node to another. We denote the expected values of a_i and z_i by $E[a_i]$ and $E[z_i]$.

4 Cooperative approach

In the cooperative approach to PHY rate and power control, we shall consider two different scenarios. In the first *max-min fair* scenario, we assign each node the same PHY rate C and MAC frame size L at all channel states. This will of course require an appropriate power control so that in bad channel conditions the transmitted power is larger. We seek to obtain the optimal PHY rate that will maximize the overall payoff of the network. As discussed before, an optimal PHY rate may not be the highest available PHY rate. In the second *global multirate* scenario, we allow each node to use a different PHY rate C_i depending on its channel conditions and we seek to obtain the globally optimal PHY rates for all nodes. In both the scenarios, it is assumed that all nodes use the same MAC frame size L. We will pursue analysis for finite n number of nodes and also consider the situation when $n \to \infty$ for both cases. We shall consider the set of possible values of C or C_i, as the case may be, to lie in an interval of the form $\underline{\mathcal{C}} := [C_l, C_u]$. In 802.11a, this interval can be $[6, 54]$. If the obtained optimal rate does not coincide with one of the values specified in the IEEE 802.11a specification i.e., one of $6, 9, ..., 54$ *Mbps* then one of the specified values which is closest to the obtained optimal rate can be used. We also assume that the cost function $Q_i(C_i)$ is convex increasing in the interval $\underline{\mathcal{C}} := [C_l, C_u]$.

4.1 Obtaining the max-min rates

A max-min assignment of resources to users is a fairness concept characterized by the property that no user i can be assigned more resources unless we decrease the assignment to another user j who already has the same amount or a lesser amount of resources than user i. This is an efficient assignment in the Pareto sense. In our case it is the PHY rates that are assigned according to the max-min approach leading to the same assignment to all users. Note that the actual throughputs are already the same for all users even if the PHY rates are different.

4.1.1 Finite number of nodes

We seek to maximize the payoff function defined in Section 3 while assigning the same PHY rate C to each node irrespective of the channel conditions. Consider the following problem:

$$\text{Find } C^* \text{ that maximizes } W_n := \sum_{i=1}^{n} (\theta(i,n) - \zeta_i Q_i(C)) \tag{13}$$

W_n is concave with respect to C (see Remark 1 for proof) and thus has a unique maximizer C^*.
In particular, we have the linear and the exponential costs as:

$$Q_i^{lin}(C) = E[a_i]C, \qquad Q_i^{exp}(C) = E[z_i]\left(e^{\psi C} - 1\right)$$

Denote

$$u^{lin} = \sum_{i=1}^{n} \zeta_i a_i, \qquad \text{and} \qquad u^{exp} = \sum_{i=1}^{n} \zeta_i z_i,$$

and therefore,

$$E[u^{lin}] = \sum_{i=1}^{n} \zeta_i E[a_i], \qquad \text{and} \qquad E[u^{exp}] = \sum_{i=1}^{n} \zeta_i E[z_i].$$

Set

$$q_1 = n\beta(1-\beta)^{n-1}L, \quad q_2 = 1 + n\beta(1-\beta)^{n-1}(T_o - T_c) + (1 - (1-\beta)^n)\, T_c. \tag{14}$$

Then we have

$$W_n^{lin}(C) = \frac{q_1}{q_2 + q_1/C} - E[u^{lin}]C \qquad W_n^{exp}(C) = \frac{q_1}{q_2 + q_1/C} - E[u^{exp}]\left(e^{\psi C} - 1\right). \tag{15}$$

Differentiating, we obtain

$$\frac{dW_n^{lin}(C)}{dC} = \frac{q_1^2}{(q_2 C + q_1)^2} - E[u^{lin}], \qquad \frac{dW_n^{exp}(C)}{dC} = \frac{q_1^2}{(q_2 C + q_1)^2} - \psi E[u^{exp}]e^{\psi C} \tag{16}$$

Equating the derivative to zero, we obtain:

(i) In the linear case, the unique positive solution of $\frac{dW_n^{lin}(C)}{dC} = 0$ is given by

$$C^* = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{E[u^{lin}]}} - 1\right) \tag{17}$$

provided that $0 < E[u^{lin}] < 1$. If $E[u^{lin}] \geq 1$ then there is no positive solution.

(ii) In the exponential case the unique positive solution of $\frac{dW_n^{exp}(C)}{dC} = 0$ is given by

$$C^* = \frac{2}{\psi} LambertW\left(\frac{1}{2}\frac{q_1}{q_2}\sqrt{\frac{\psi}{E[u^{exp}]}}\exp\left(\frac{1}{2}\frac{q_1}{q_2}\psi\right)\right) - \frac{q_1}{q_2} \tag{18}$$

(iii) In either the linear or the exponential case, if C^* lies within $\underline{\mathcal{C}}$ then it is the unique globally optimal rate assignment solution for problem (13). If not, then the optimal solution is obtained on one of the two boundary points of $\underline{\mathcal{C}}$.

We defer the discussion on the numerical computations of C^* to Section 6.

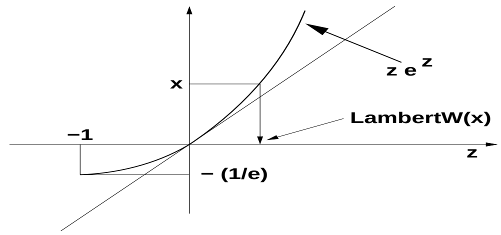

Figure 1: The LambertW function is the inverse function of ze^z; notice that for $x \geq -\frac{1}{e}$, $LambertW(x) \leq x$, with equality only for $x = 0$.

4.1.2 The asymptotic case

We present below the asymptotic behaviour for large number of mobile nodes. Our optimization will be based on the expression for the asymptotic throughput given by Equation (3). Here we assume that a_i, z_i and ζ_i have the same distribution for all mobiles. Consider the following problem:

$$\text{Find } C^* \text{ that maximizes } W(C) := \tau(C) - \zeta Q(C) \tag{19}$$

where $Q(C) = E[a]C$ for the linear cost and $Q(C) = E[z]\left(e^{\psi C} - 1\right)$ for the exponential one.

As we shall see later that $W(C)$ turns out to be concave in C (where we make use of the inequality (2)) and therefore it has a unique maximizer. Writing $W(C)$ for the linear and exponential case as

$$W^{lin}(C) = \frac{q_1}{q_2 + q_1/C} - E[a]C \quad \text{and} \quad W^{exp}(C) = \frac{q_1}{q_2 + q_1/C} - E[z]\left(e^{\psi C} - 1\right)$$

where

$$q_1 = L\left(1 - \frac{1}{p}\right), \qquad q_2 = \frac{1 + T_c/p}{\ln\left(\frac{p}{p-1}\right)} + \left(1 - \frac{1}{p}\right)(T_o - T_c). \tag{20}$$

Then the optimal C is obtained by differentiating $W^{lin}(C)$ and $W^{exp}(C)$ and equating them to 0. For the linear case this gives

$$\frac{dW^{lin}(C)}{dC} = \frac{q_1^2}{(q_2 C + q_1)^2} - \zeta E[a] = 0 \tag{21}$$

For the exponential case we have:

$$\frac{dW^{exp}(C)}{dC} = \frac{q_1^2}{(q_2 C + q_1)^2} - \psi\zeta E[z]e^{\psi C} = 0 \tag{22}$$

Rearranging Equations (21) and (22) we obtain the following result:

(i) In the linear case, the unique positive solution of (21) is given by

$$C^* = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{\zeta E[a]}} - 1\right).$$

(ii) In the exponential case, the unique positive solution of (22) is given by

$$C^* = \frac{2}{\psi} LambertW\left(\frac{1}{2}\frac{q_1}{q_2}\sqrt{\frac{\psi}{\zeta E[z]}} \exp\left(\frac{1}{2}\frac{q_1}{q_2}\psi\right)\right) - \frac{q_1}{q_2}$$

(iii) If C^* lies within $\underline{\mathcal{C}}$ then it is the unique globally optimal rate assignment solution for problem (19). If not then the optimal solution is obtained on one of the two boundary points of $\underline{\mathcal{C}}$.

Note that C^* here has the same form as in the finite n case but with different q_1 and q_2.

Remark 1 *Consider the linear and the exponential payoff functions:*
(i) The second derivative of the linear and exponential payoffs is given by

$$-\frac{2q_1^2 q_2}{(q_2 C + q_1)^3} \qquad \textit{and} \qquad -\frac{2q_1^2 q_2}{(q_2 C + q_1)^3} - \psi^2 \zeta E[z] e^{\psi C}$$

respectively, where q1 and q2 for finite n and $n \to \infty$ are given by Equations (14) and (20) respectively. The second derivative is strictly negative by the assumption made in Equation (2) and hence the payoff functions considered in the foregoing discussion are indeed (strictly) concave.
(ii) Equation (22) has another solution given by

$$C^o = 2LambertW\left(-\frac{1}{2}\frac{q_1}{q_2}\frac{\exp\left(\frac{1}{2}\frac{q_1}{q_2}\right)}{\sqrt{\zeta E[z]}}\right) - \frac{q_1}{q_2}$$

However, since $Lambert(x) < 0$ for all $x < 0$, C^o is not in $\underline{\mathcal{C}}$.

4.2 Computing QoS for the dynamic case

So far we have considered a fixed number of mobile nodes in the system. In this section we consider a dynamic setting.

Let mobiles arrive in the WLAN system according to an independent Poisson process with rate λ. The nth mobile is assumed to have a service requirement σ_n where σ_n are i.i.d. generally distributed. We assume that the assignment of physical rates follows the max-min fairness approach, so that the physical rate of each mobile node is given by $C(n)$, where n is the number of mobiles in the system; n can be referred to as the system state. Then the throughput of each mobile when the system is in state n is given by

$$\theta(n) = \frac{\beta(1-\beta)^{n-1}L}{1 + n\beta(1-\beta)^{n-1}\left(T_o - T_c + \frac{L}{C}\right) + \Big(1 - (1-\beta)^n\Big)T_c} \tag{23}$$

Since all mobiles are assigned the same rate by the max-min fairness approach and hence theoretically achieve the same throughput, the whole network can be viewed as an $M/G/1/\infty$ queue where the service discipline is a generalized processor sharing (GPS) [1]. If we denote $\rho := \lambda E[\sigma_0]$ and define

$$\phi(n) = \frac{1}{\prod_{i=1}^{n}\theta(i)}.$$

then applying the general theory of [1] for GPS queues, we obtain the following expressions for the steady state probabilities and sojourn times:

Theorem 4.1 *Assume that*

$$\sum_{i=1}^{\infty}\frac{\rho^i}{i!}\phi(i) < \infty.$$

Then
(i) the steady state probabilities exist and are given by

$$Pr(N = n) = \frac{\rho^n \phi(n)}{n! \sum_{i=0}^{\infty}\frac{\rho^i}{i!}\phi(i)}.$$

(ii) The expected sojourn time of a mobile is given by:

$$E[T] = E[\sigma]\frac{\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\phi(j+1)}{\sum_{j=0}^{\infty}\frac{\rho^j}{j!}\phi(j)}$$

Remark 2 *Using [1] one can in fact*
(i) obtain explicit expressions for the QoS of more complex arrival process, and in particular for on/off sources having general thinking times.
(ii) obtain explicit expressions for the case when there is a limit (enforced by a call admission control) on the number of active mobiles.

4.3 Global multirate (channel dependent) optimization

In this section we consider the global optimization in which we allow each node to use a different PHY rate C_i and we seek to obtain the best choice of C_i, $i = 1, ..., n$. We assume that all values of $E[a_i]$, $E[z_i]$ and ζ_i are known by the decision maker. By allowing C_i to differ from one node to another we expect to achieve higher efficiency.

4.3.1 Finite number of nodes, channel-dependent case

Consider the following problem:

$$\text{Find } \mathbf{C}^* = (C_1^*, ..., C_n^*) \text{ that maximizes } W_n := \sum_{i=1}^{n}(\theta(i,n) - \zeta_i Q_i(C_i)) \tag{24}$$

where $\theta(i, n)$ is defined by Equation (1). Then we have,

$$W_n^{lin} = \frac{q_1}{q_2 + \frac{q_1}{n}\sum_{i=1}^{n}(\frac{1}{C_i})} - \sum_{i=1}^{n} \zeta_i E[a_i] C_i \tag{25}$$

and

$$W_n^{exp} = \frac{q_1}{q_2 + \frac{q_1}{n}\sum_{i=1}^{n}(\frac{1}{C_i})} - \sum_{i=1}^{n} \zeta_i E[z_i](e^{\psi C_i} - 1) \tag{26}$$

where q_1 and q_2 are defined in (14). Let

$$q_2^i := q_2 + \frac{q_1}{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{C_j} \tag{27}$$

Denote by $\hat{C}$ the harmonic average of C_i's, i.e.,

$$\hat{C} = \left(\sum_{i=1}^{n} \frac{1}{C_i} \right)^{-1} \tag{28}$$

With these definitions and by differentiating (25) we get

$$\frac{\partial W_n^{lin}}{\partial C_i} = \frac{nq_1^2}{(nq_2^i C_i + q_1)^2} - \zeta_i E[a_i] = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 nC_i^2} - \zeta_i E[a_i] \tag{29}$$

and similarly by differentiating (26)

$$\frac{\partial W_n^{exp}}{\partial C_i} = \frac{nq_1^2}{(nq_2^i C_i + q_1)^2} - \psi\zeta_i E[z_i]e^{\psi C_i} = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 nC_i^2} - \psi\zeta_i E[z_i]e^{\psi C_i} \tag{30}$$

Denote

$$H(\hat{C}) = \frac{q_1^2}{\left(q_2 + \frac{q_1}{n\hat{C}}\right)^2 n} \tag{31}$$

Now by equating the derivatives in Equations (29) and (30) to zero, we obtain:

(i) In the linear case,

$$C_i = \sqrt{\frac{H(\hat{C})}{\zeta_i E[a_i]}} \tag{32}$$

Let

$$Y := \sum_{i=1}^{n} \sqrt{\zeta_i E[a_i]} \tag{33}$$

Then

$$\hat{C} = \left(\sum_{i=1}^{n} \sqrt{\frac{\zeta_i E[a_i]}{H(\hat{C})}} \right)^{-1} = \frac{\sqrt{H(\hat{C})}}{Y}.$$

which implies that the solution $\hat{C}^*$ is given by

$$\hat{C}^* = \frac{1}{n} \frac{q_1}{q_2} \left(\frac{\sqrt{n}}{Y} - 1 \right)$$

Substituting the solution of this equation in Equation (32) gives the C_i^*'s.

(ii) In the exponential case, we get from Equation (30)

$$C_i = \frac{2}{\psi} LambertW \left(\frac{1}{2} \sqrt{\frac{\psi H(\hat{C})}{\zeta_i E[z_i]}} \right). \tag{34}$$

Therefore, using (28) it implies that $\hat{C}^*$ is the solution of

$$\hat{C} = \frac{2}{\psi \sum_{i=1}^{n} \left[LambertW \left(\frac{1}{2} \sqrt{\frac{\psi H(\hat{C})}{\zeta_i E[z_i]}} \right) \right]^{-1}}$$

which yields the C_i^*'s through Equation (34).

The above solutions are globally optimal provided they are within the range $\underline{\mathcal{C}}$. We defer the discussion on the numerical computations of C_i^*'s to Section 6.

4.3.2 Large number of nodes

To model the case of a large number of users we shall use a fluid approximation in which there are (non-countably) infinite number of users. (This type of approach is frequently used in other engineering fields, see e.g. [6].) We introduce R population classes of mobiles. The parameter z in the cost function (11) will be the same for all mobiles of the same type $r, r = 1, 2, ..., R$ so that mobiles belonging to a given class r have the same channel conditions. We shall thus use the notation $z^{(r)}$ to indicate this dependence. We shall use similarly the notation $a^{(r)}$ for the coefficient appearing in the linear cost. In short, mobiles with the same value of $(a^{(r)}, \zeta^{(r)})$ (in the linear case) or $(z^{(r)}, \zeta^{(r)})$ (in the exponential case) are said to belong to the same class of mobiles having identical channel conditions.

We define for each r the vector $\mathbf{x}^{(r)} = (x_1^{(r)}, ..., x_c^{(r)})$ to be the amount of r-type mobiles that use each of the rates $C_1, ..., C_c$. Define $\mathbf{x} = (\mathbf{x}^{(1)}, ..., \mathbf{x}^{(R)})$ to be a multistrategy for all mobiles. With some abuse of notation, let $x_i := \sum_{r=1}^{R} x_i^{(r)}$ denote the global amount of mobiles that use the rate C_i under $\mathbf{x}$. Denote $\overline{\nu}$ to be the total amount of users. Then $\overline{\nu} = \sum_{i=1}^{c} x_i$. Define $\alpha_i(\mathbf{x}) = x_i/\overline{\nu}$. It follows from Equation (7) that

$$\tau(\alpha(\mathbf{x})) = \frac{E_{\alpha(\mathbf{x})}[1] q_1}{q_2 + q_1 E_{\alpha(\mathbf{x})}[1/C]} = \frac{\overline{\nu} q_1}{\overline{\nu} q_2 + q_1 \sum_{i=1}^{c} x_i C_i^{-1}}$$

where q_1 and q_2 are given by Equation (20). To simplify, we shall denote $\tau(\mathbf{x}) = \tau(\alpha(\mathbf{x}))$. Define $Q_i^{(r)}(x_i^{(r)}) = a^{(r)} C_i$ for the linear cost and $Q_i^{(r)}(x_i^{(r)}) = z^{(r)} \left(e^{\psi C_i} - 1 \right)$ for the exponential cost.

Then our problem of maximizing the payoff function turns out to be a non-linear optimization problem defined by:

$$\begin{aligned}\max_{\mathbf{x}} W(\mathbf{x}) \quad \text{where } W(\mathbf{x}) &:= \tau(\mathbf{x}) - \sum_{r=1}^{R} \zeta^{(r)} \sum_{i=1}^{c} x_i^{(r)} Q_i^{(r)}(x_i^{(r)}) \\ &= \frac{\overline{\nu} q_1}{\overline{\nu} q_2 + q_1 \sum_{i=1}^{c} \left(\sum_{r=1}^{R} x_i^{(r)}\right) C_i^{-1}} - \sum_{r=1}^{R} \zeta^{(r)} \sum_{i=1}^{c} x_i^{(r)} Q_i^{(r)}(x_i^{(r)}) \\ \text{subject to} \quad & \sum_{i=1}^{c} x_i^{(r)} = g_r, \forall r, \quad x_i^{(r)} \geq 0, \forall i, r\end{aligned}$$

where g_r is the predefined constraint on the number of mobiles in class r. Describing the solution for this problem is outside the scope of this report. However, $W(\mathbf{x})$ turns out to be concave and the feasible set is compact. We conclude that there exists a unique solution.

5 Non-cooperative game

5.1 Finite number of nodes

In this section we analyse the non-cooperative behaviour of mobile nodes. We shall model this situation using non-cooperative game theory and obtain the equilibrium.

In a non-cooperative setting, each node uses the same MAC frame size L and is allowed to use a different PHY rate as in the global multirate allocation in the cooperative approach. But here the objective of each node is to maximize its individual payoff function which can be denoted by $\Omega_i(C_i)$ and defined as

$$\Omega_i(C_i) = \theta(i, n) - \zeta_i Q_i(C_i) \tag{35}$$

For every i, Ω_i is concave w.r.t. C_i and continuous w.r.t. $C_j, j \neq i$. It then follows from Rosen [7] that a Nash equilibrium exists. In particular, we shall be interested in the linear $Q_i^{lin}(C_i) = E[a_i]C_i$ and the exponential $Q_i^{exp}(C_i) = E[z_i]\left(e^{\psi C_i} - 1\right)$ cases.

We have

$$\frac{\partial \Omega_i^{lin}}{\partial C_i} = \frac{H(\hat{C})}{n C_i^2} - \zeta_i E[a_i], \qquad \frac{\partial \Omega_i^{exp}}{\partial C_i} = \frac{H(\hat{C})}{n C_i^2} - \psi \zeta_i E[z_i] e^{\psi C_i} \tag{36}$$

These are the same equations as we had in Section 4.3 except with an extra factor of n in the denominator. Equating them to zero:

(i) For the linear case we obtain

$$\hat{C}^* = \frac{1}{n} \frac{q_1}{q_2} \left(\frac{1}{Y} - 1\right) \tag{37}$$

where Y is the same as defined in Equation (33) and C_i^*'s are obtained from

$$C_i = \sqrt{\frac{H(\hat{C})}{n \zeta_i E[a_i]}}. \tag{38}$$

(ii) Similarly for the exponential case, we obtain $\hat{C}^*$ as a solution of

$$\hat{C} = \frac{2}{\psi \sum_{i=1}^{n} \left[LambertW \left(\frac{1}{2} \sqrt{\frac{\psi H(\hat{C})}{n \zeta_i E[z_i]}} \right) \right]^{-1}}$$

which yields the C_i^*'s through

$$C_i = \frac{2}{\psi} LambertW \left(\frac{1}{2} \sqrt{\frac{\psi H(\hat{C}^*)}{n \zeta_i E[z_i]}} \right). \tag{39}$$

We defer the discussion on the numerical computations of C_i^*'s to Section 6.

5.2 Non-cooperative game with infinitely many users

In the case of a very large number of players, a reasonable model is of a game with a continuum set of players. We are motivated by the Wardrop equilibrium concept in road traffic (in which routes of cars are chosen so as to minimize the cars' delays, [9, 6], and where the set of cars is assumed to be infinite).

If we denote the throughput of all nodes in class r by $\tau_r(\mathbf{x})$ and the payoff of a node in class r by $W_r(\mathbf{x}, C')$ then for the exponential power consumption costs, the payoff is given by

$$W_r(\mathbf{x}, C') := \frac{1}{g_r}\tau_r(\mathbf{x}) - \zeta^{(r)} z^{(r)} \left(e^{\psi C'} - 1\right) \tag{40}$$

where g_r is the same as defined in Section 4.3.2. Now, $\mathbf{x}^* = (\mathbf{x}^{(1)*}, ..., \mathbf{x}^{(R)*})$ is called a Wardrop type equilibrium if for each r we have

$$(x_i^{(r)})^* \geq 0, \forall i, r \qquad \sum_{i=1}^{c} (x_i^{(r)})^* = g_r, \forall r, \qquad W_r(\mathbf{x}^*, C') \geq W_r(\mathbf{x}^*, C),$$

where the last inequality holds for all $C \in \underline{\mathcal{C}}$, and for all C' in the support of $\mathbf{x}^{(r)*}$ (i.e., for all i for which $((x_i^{(r)})^* > 0)$.

It is seen from (40) that for any value $\mathbf{x}$, the maximization of $W_r(\mathbf{x}, C')$ over C' is obtained by choosing the smallest available C'. This implies that as the number of nodes becomes large, the only Wardrop equilibrium that is highly inefficient is when: all nodes end up using the smallest available physical rate. The reason behind it is that in the case of a lot of users, the choice of C' by a single node has a negligible impact on its own throughput, and hence the node only considers minimizing its cost related to the power consumption.

6 Numerical Studies

In this section, we numerically examine the closed form expressions for the optimal transmission rates obtained in the previous sections. We also examine the corresponding single node throughputs and overall payoffs. We compute the optimal PHY transmission rates, throughputs and payoffs as a function of number of nodes n and also as a function of the MAC frame size L. Since we consider C or C_i, as the case may be, to lie in the continuous interval $\underline{\mathcal{C}} := [C_l, C_u]$, we don't get C or C_i's to be a set of discrete values that can be directly assigned to nodes as per the IEEE 802.11 specifications. Consequently, if the obtained optimal rate C or C_i does not coincide with one of the discrete values specified in the IEEE 802.11a specification i.e., one of $6, 9, ..., 54$ $Mbps$, then a discrete specified value which is closest to the obtained optimal rate can be used.

We use the following set of parameters to study the optimal transmission rates and the corresponding single node throughputs and overall payoffs:

- In the linear cost $E[a_i]$ is set to vary uniformly from $a_i^{min} = 0.5 * 10^{-3}$ to $a_i^{max} = 1 * 10^{-3}$ $watts$ per $bits/slot$ for each mobile i.
- In the exponential cost $E[z_i]$ is set to vary uniformly from $z_i^{min} = WN_o/h_i^{min}$ to $z_i^{max} = WN_o/h_i^{max}$ where value of W (passband spectrum) is taken as 20 MHz for an 802.11a system, N_o (one-sided power spectral density) is taken as $5.52 * 10^{-21}$ $watts/Hz$ and for the Rayleigh fading case $h_i^{min} = 10^{-11}$ and $h_i^{max} = 10^{-8}$.
- The back-off multiplier $p = 2$ and $b_0 = 16$ slots in $b_k = p^k b_0$.
- The data frame transmission overhead $T_o = 52$ slots and the RTS collision overhead $T_c = 17$ slots.
- The slot size is taken as $20\mu s$ and $K = 10$ [5].
- For simplification the parameter ζ_i is taken to be the same for all nodes $i = 1..n$. The values are displayed below each plot.

The plots obtained from the numerical computations are presented at the end of the report. The first set of plots (Figure 1 to 12) are variations of optimal PHY rates, single node throughput and overall payoff w.r.t. number of nodes n and the second set of plots (Figure 13 to 24) are variations of optimal PHY rates, single node throughput and overall payoff w.r.t. the MAC frame size L.

Comparison between cooperative and non-cooperative solutions: In the PHY transmission rate plots (Figure 2- 6 and 8- 12), we observe that the optimal PHY rate of each node decreases with increasing number of nodes. It can be seen that in the cooperative global multirate and non-cooperative multirate allocations, for a given number of nodes, each node is assigned a different rate depending on the parameters $E[a_i]$ and $E[z_i]$ for channel conditions. The curvature along the "node index" axis is more significant in the exponential case than in the linear case. This is due to the inherent form of the solution for the exponential case that consists of a "LambertW" function representation.

When we have a *linear* cost associated with the power consumption, then for $n = 2$, the single node throughput in the cooperative global multirate case (Figure 5) is around 11% higher than in the non-cooperative multirate case (Figure 7). In fact with increasing n the single node throughput percentage gain in the cooperative global multirate scenario over the non-cooperative multirate scenario goes from 11% for $n = 2$ to up to more than 200% for $n = 10$. When the cost associated with the power consumption is *exponential* the single node throughput percentage gain in the cooperative allocation (Figure 11) over the non-cooperative allocation (Figure 13) varies from around 12% for $n = 2$ to up to 100% for $n = 10$. We also observe that the cooperative max-min fair scheme (Figure 3, 9) performs almost equally well as the cooperative global multirate scheme (Figure 5, 11).

These observations clearly illustrate that cooperative PHY rate allocation strategy results in higher single node throughputs and hence higher total network throughput as against a non-cooperative strategy. Our analysis thus confirms the results obtained by Tan et al. in [12]. Indeed the DCF protocol under a non-cooperative setting is not efficient.

In the plots for variation w.r.t. the MAC frame size (Figure 15, 17, 19, 21, 23 and 25), we observe for $n = 10$ that the single node throughput increases with increasing frame size. This is an expected behaviour due to the fact that the obtained optimal PHY rates increase with increasing frame size.

7 Conclusion and Future Work

In this report we have analysed cooperative and non-cooperative rate and power control in an IEEE 802.11 WLAN. Our analysis is based on a throughput expression derived in [5]. We seek to obtain optimal PHY rate assignments in both cooperative and non-cooperative scenario by optimizing a payoff function that comprises of the throughput and costs related to power consumption. We consider two different types of cost functions–linear and exponential. In the cooperative approach, we maximize the overall payoff of the network which inherently maximizes the total network throughput and minimizes costs related to the power consumption of all nodes. The focus in non-cooperative game analysis is to maximize individual node's payoff. Our main contribution in this report is that we obtain explicit expressions for the optimal PHY rates. It is observed through numerical studies that cooperative control is more efficient than non-cooperative control. With a linear cost approximation, the single node throughput in the cooperative approach is observed to be 11% to 200% more than in the non-cooperative game approach. The improvement varies from 12% to 100% in the exponential cost approximation case. Thus a first glimpse of cooperative and non-cooperative control in an 802.11 WLAN by our analysis shows that the currently used mandatory DCF protocol in 802.11 does not perform with the highest efficiency in a non-cooperative setting.

Our future work will include designing efficient cooperative rate and power control algorithms based on the analysis illustrated in this report. The algorithm should be *distributed* in the lines of DCF so that it can be used in both ad-hoc and infrastructure networks.

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Variation with n and linear cost

Cooperative Approach

Max-min fair

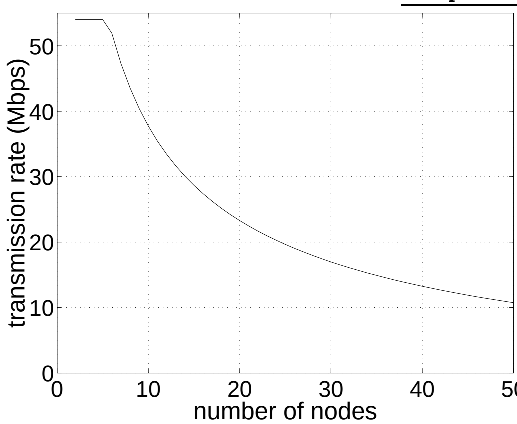

Figure 2: Using Equation 17, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = 6$

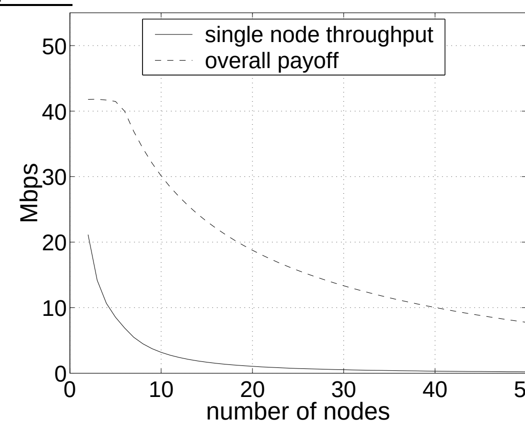

Figure 3: Using Equation 15, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = 6$

Global multirate

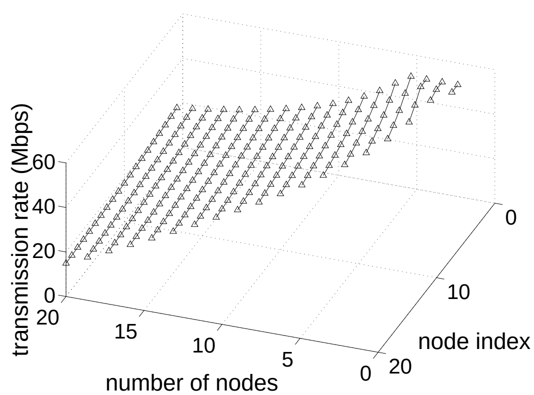

Figure 4: Using Equation 32, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = 9$

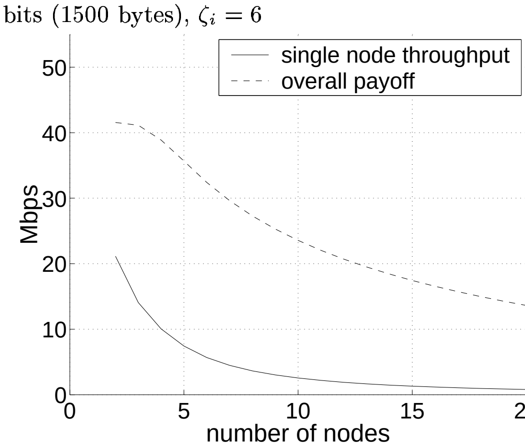

Figure 5: Using Equation 25, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = 9$

Non-cooperative game

Multirate

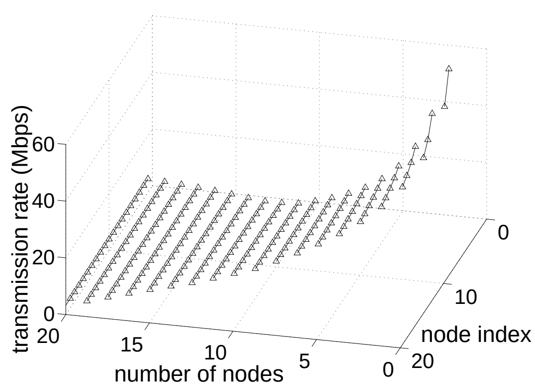

Figure 6: Using Equation 38, frame size $L = 12000$ bits (1500 bytes),$\zeta_i = 9$

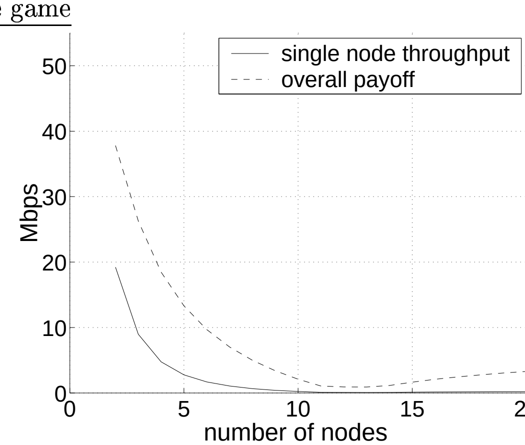

Figure 7: Using Equation 35, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = 9$

Variation with n and exponential cost

Cooperative Approach

Max-min fair

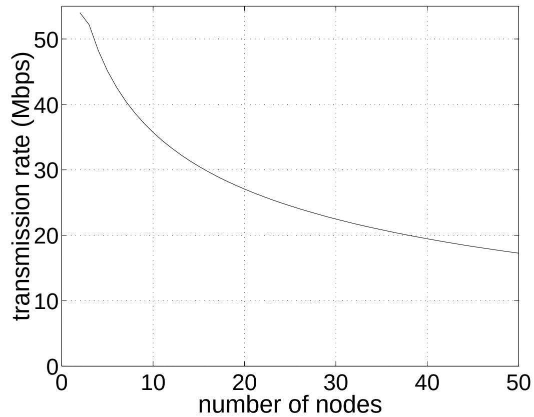

Figure 8: Using Equation 18, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = e^5$

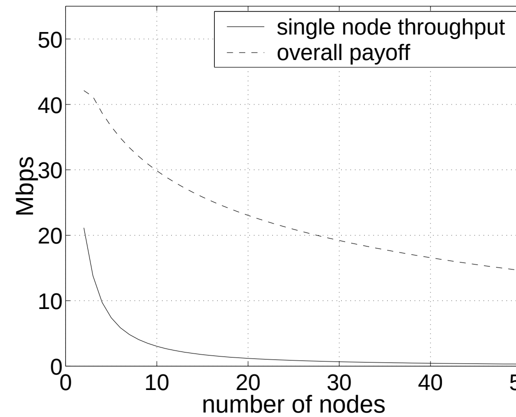

Figure 9: Using Equation 15, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = e^5$

Global multirate

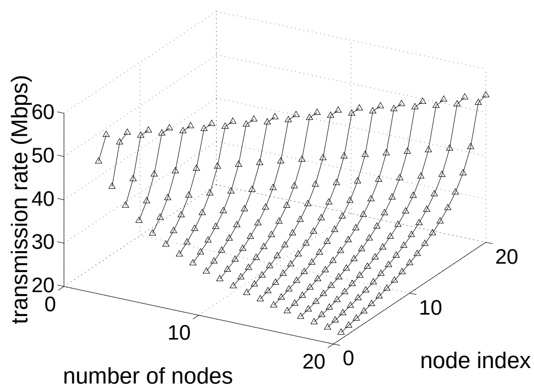

Figure 10: Using Equation 34, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = e^5$

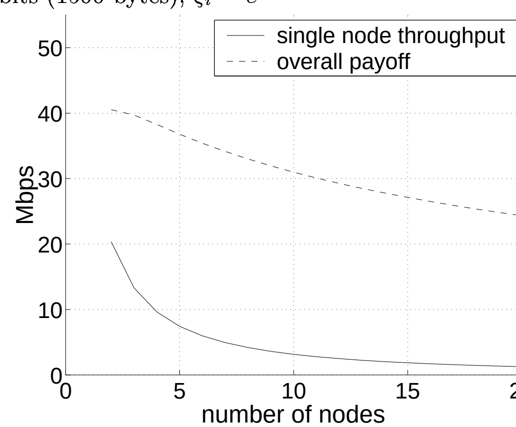

Figure 11: Using Equation 26, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = e^5$

Non-cooperative game

Multirate

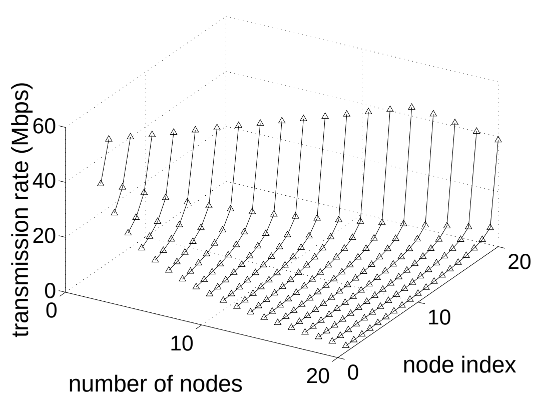

Figure 12: Using Equation 39, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = e^5$

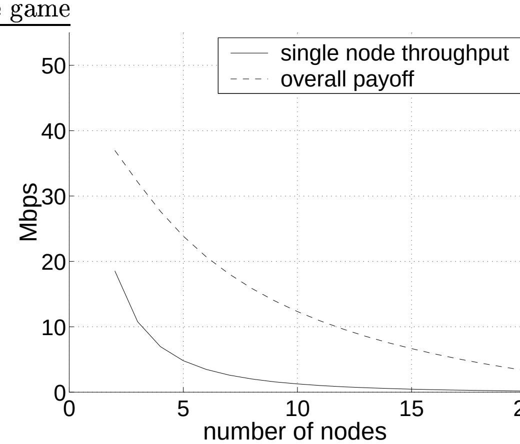

Figure 13: Using Equation 35, frame size $L = 12000$ bits (1500 bytes), $\zeta_i = e^5$

Variation with L and linear cost

Cooperative Approach

Max-min fair

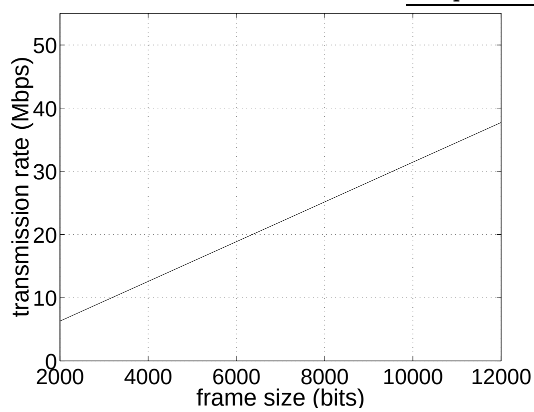

Figure 14: Using Equation 17, $\zeta_i = 6$

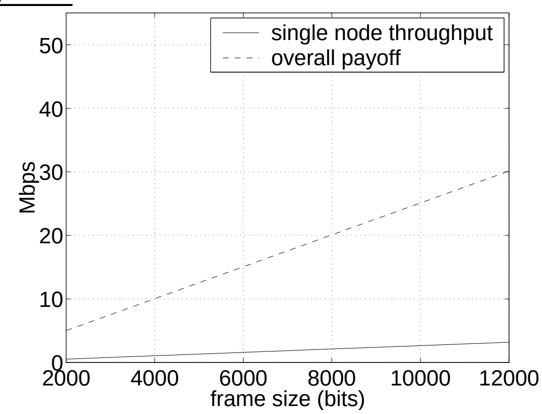

Figure 15: Using Equation 15, $\zeta_i = 6$

Global multirate

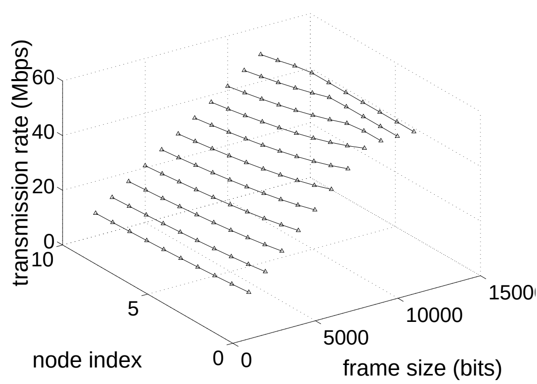

Figure 16: Using Equation 32, $\zeta_i = 3$

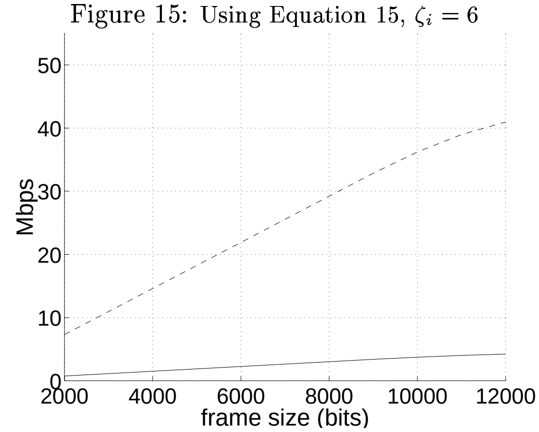

Figure 17: Using Equation 25, $\zeta_i = 3$

Non-cooperative game

Multirate

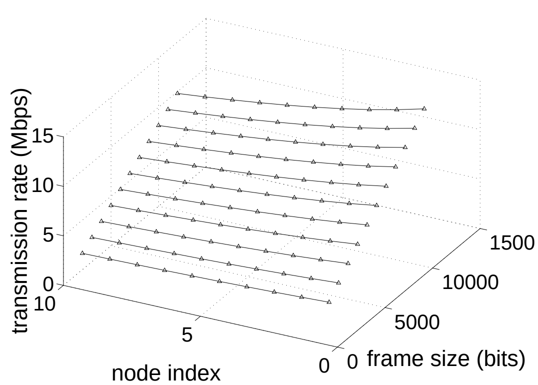

Figure 18: Using Equation 38, $\zeta_i = 3$

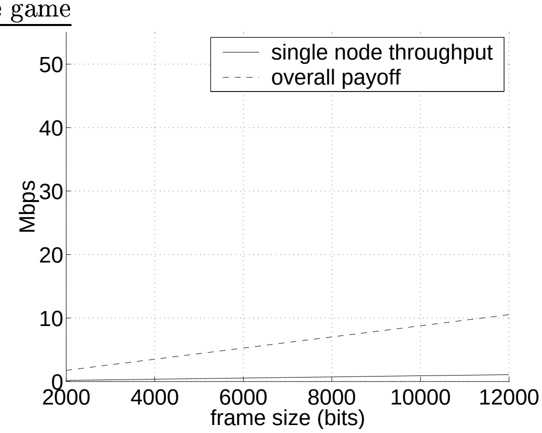

Figure 19: Using Equation 35, $\zeta_i = 3$

Variation with L and exponential cost

Cooperative Approach

Max-min fair

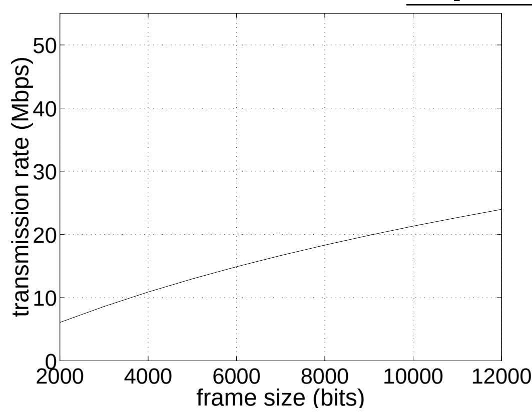

Figure 20: Using Equation 18, $\zeta_i = e^6$

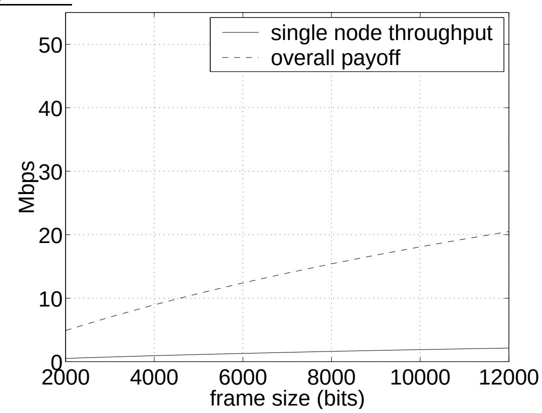

Figure 21: Using Equation 15, $\zeta_i = e^6$

Global multirate

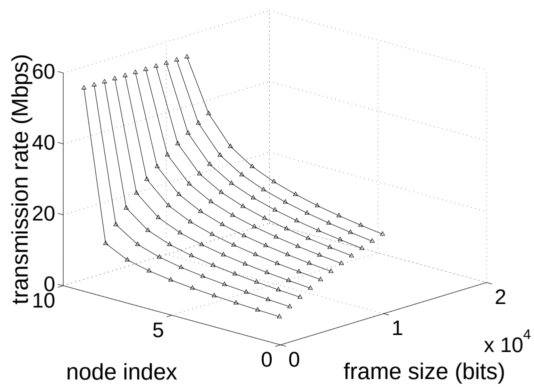

Figure 22: Using Equation 34, $\zeta_i = e^6$

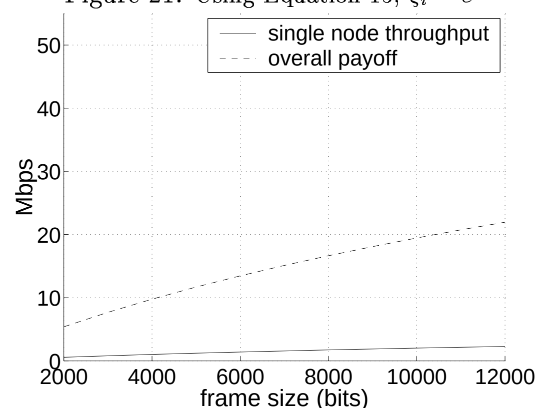

Figure 23: Using Equation 26, $\zeta_i = e^6$

Non-cooperative game

Multirate

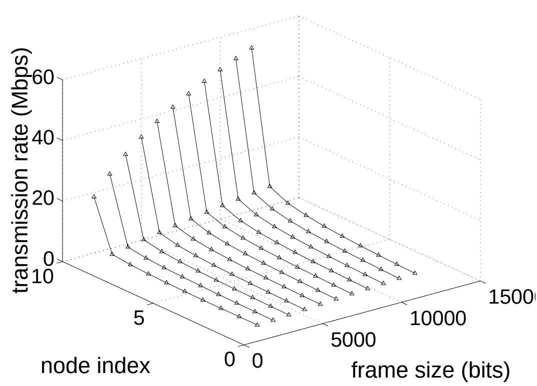

Figure 24: Using Equation 39, $\zeta_i = e^6$

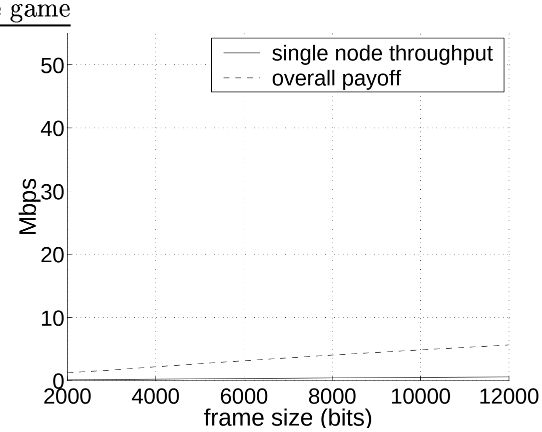

Figure 25: Using Equation 35, $\zeta_i = e^6$

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